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EFFECTS OF A MAGNETIC FIELD ON THE DIELECTRIC CONSTANT AND CONDUCTIVITY OF 2D METALS*.

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An exact formula for the RPA dielectric function of a 2D electron gas in a strong magnetic field is given explicitly as a function of the field strength, frequency and momentum. Based on this result, important many body effects in 2D conductors are discussed. The magnetic susceptibility is obtained explicitly. It shows characteristic oscillations, cyclotron resonance, and an interesting temperature variation. The carrier and field dependences of the effective mass and scattering time of the 2D electrons are evaluated in terms of the memory function formalism. Our theoretical results are found generally in very good agreement with recent data on Si inversion layers.

1. INTRODUCTION

Chalcogenide compounds, MX_2, of vanadium, niobium and tantalum are known to be two-dimensional. The atoms in these compounds are strongly bonded into layers of three atomic thickness, sandwiched between two close packed sheets of

chalcogenides such as sulfur, selenium or tellurium. A typical dichalcogenide, $NbSe_2$ is metallic, shows incommensurate charge-density waves and becomes a superconductor below 7°K.[1] Together with the one-dimensional counterpart $NbSe_3$, it constitutes a very interesting low-dimensional system.

There are other two-dimensional systems. The electrons in inversion or accumulation layers of MOSFETs, at the interface of epitaxially grown GaAs-GaAlAs heterojunctions or on the surface of liquid helium are known to be two-dimensional and show prominent many body effects in wide density ranges.[2]

One of the very useful experimental techniques for two-dimensional electron systems is a tilted magnetic field method. When the field is tilted from the direction perpendicular to the surface in which the electrons move, the electron's orbital energy is dependent on the tilted angle but the spin-field coupling is independent.[3] Hence, the two couplings can be separated effectively.

On the other hand, for the theoretical study of the electronic properties of various conductors, the dielectric function plays an important role. It determines conductivity, cyclotron resonance, reflectance, etc.

Based on these observations, we shall report in the next section an explicit RPA dielectric function of a two-dimensional electron gas in a strong magnetic field. Our formula is reduced exactly to the known expression in the absence of the field. The static dielectric function for this case has been illustrated for one, two, and three dimensions.[4] We shall then make use of the dielectric function for the evaluation of the susceptibility, effective mass and scattering time of 2D electrons. For simplicity, we shall use the natural unit in which $\hbar = 1$ and $2m = 1$ unless when their

explicit display is desirable.

2. DIELECTRIC FUNCTION

For our purpose, we find it advantageous to use the eigenvalues of the electron propagator representing the unit of the so-called ring diagrams. As shown elsewhere,[5] the eigenvalues $\lambda_j(q)$ yield the density response function when $2\pi j/\beta$ is replaced by $-i\omega$ where $\beta = 1/kT$. The dielectric function $\varepsilon(q,\omega)$ is then given by

$$\varepsilon(q,\omega) = 1 + \lambda(q,\omega)\, u(q) \tag{2.1}$$

where $u(q)$ is the Coulomb interaction. For two dimensions, it is $2\pi e^2/q$. The evaluation of the eigenvalues under the dHvA condition has been made recently by Isihara and Kojima based on some approximations.[6] We have succeeded in eliminating such approximations to arrive at a low temperature formula which is essentially exact towards absolute zero:

$$\varepsilon(q,\omega) = \varepsilon^{o}(q,\omega) + \frac{2\gamma}{\eta}\, u(q)\, I(q,\omega) \times \sum_s \frac{(-)^s}{\sinh(\pi^2 s/\alpha)} \sin\frac{\pi s}{\gamma} \cos(\tfrac{1}{2} g\pi s) \tag{2.2}$$

where $\varepsilon^{o}(q,\omega)$ is the known dielectric function in the absence of the magnetic field,[4] $\gamma = \omega_o/2p_F^2$, $\alpha = \beta\omega_o/2$, g is the g factor, ω_o is the cyclotron frequency, and the amplitude function is given by

$$I(q,\omega) = [1 - e^{2\pi i\omega/\omega_o}]^{-1} \times \int_0^{2\pi} dx\, \exp(i\omega x/\omega_o) \sin(Q\sin x) J_o(X) \tag{2.3}$$

where $Q = q^2/\omega_o$, $X = 2[Q(1-\cos x)/\gamma]$.

Formula (2.2) gives the dielectric function explicitly as a function of the magnetic field under the dHvA conditions. It shows characteristic oscillations and cyclotron resonance. It is a Sommerfeld type low temperature expression which has been obtained to order r_s under the dHvA conditions. Note that it is independent of Coulomb interaction. For the static case, it is given for small q as follows:

$$\varepsilon(q,0) = 1 + e^2[R^+J_1(2R^+) + R^-J_1(2R^-)]/2q \qquad (2.4)$$

where

$$R^{\pm} = 2^{1/2}qp_F^{\pm}/\omega_o,\ p_F^{\pm} = \{p_F^2 \pm (g/4)\omega_o\}^{1/2}$$

We have investigated numerically the real and imaginary parts of the field independent dielectric function $\varepsilon^o(q,\omega)$ as functions of the reduced frequency $\omega/2p_F^2$ for $r_s = 1$. For small q, the real part is found to be rather large in the limit $\omega\to 0$, indicating a strong screening. The imaginary part vanishes at the origin and is finite in a limited region of frequency which depends on q. For instance, for $q = 0.5p_F$ it is finite only up to around $1.3\ p_F^2$.

3. SUSCEPTIBILITY

The advantage of our present approach is that a sum rule of the eigenvalues yields the exchange grand partition function immediately.

The magnetic susceptibility is then found as follows:

$$\frac{\chi}{\chi_o} = \frac{2}{\alpha}\sum_{s=1}^{\infty}(-)^{s+1}\frac{\cos(g\pi s/2)}{s\sinh(\pi^2 s/2)} \times \{[1 + \frac{\pi^2 s}{\alpha}\coth(\frac{\pi^2 s}{\alpha})]\cos\frac{\pi s}{\gamma_o} + \frac{\pi s}{\gamma_o}\sin(\frac{\pi s}{\gamma_o})\} \qquad (3.1)$$

where $\chi_o = e^2/2\pi c^2$ is the ideal gas susceptibility and and $\alpha = \beta\mu_B H$. In the ordinary unit, χ_o is $(4\pi m/h^2)\mu_B^2$.

Note that χ is explicitly given as a funcion of density n through $\gamma_o = \omega_o/2p_o^2$, $p_o^2 = 2\pi n$.

As a 2D characteristic, the susceptibility has a dimension. It oscillates with the amplitude which varies with the field as H^{-2}. The oscillation is determined by the ratio of the Fermi energy to the field energy. Formula (3.1) is exact to order r_s and yet does not have r_s. This is a 2D peculiarity. At high temperatures, one can show that the susceptibility approaches Curie's law.

4. CONDUCTIVITY

The dielectric function shows infinitely sharp cyclotron resonance. In actual 2D systems, electrons are scattered by impurities, causing broadening of the Landau levels. We then expect Lorentzian peaks due to the electron self energy Σ_n because the denominator will be of the form $(\omega - n\omega_o - \Sigma_n)$ where n is an integer.

According to the memory function formalism,[7] the conductivity is determined by the memory function $M(\omega)$ which in turn is a function of the dielectric function. By assuming the Drude form, we then find the effective mass and relaxation time as follows:

$$\sigma(\omega) = \frac{ine^2}{m}\{\omega - \omega_o + M(\omega)\}^{-1} \tag{4.1}$$

where the memory function $M(\omega)$ is given by

$$M(\omega) = \frac{n_i}{4\pi mn}\int dq\, q^3 \frac{v^2(q)}{u(q)}\left\{\frac{1}{\varepsilon(q,0)} - \frac{1}{\varepsilon(q,\omega)}\right\} \tag{4.2}$$

Here, n_i is the impurity density and $v(q)$ is the impurity potential. The cyclotron effective mass m^* and scattering time are then given by

$$\frac{m^*}{m} = 1 + \frac{M_1}{\omega}, \quad \frac{1}{\tau} = M_2(\omega)\{1 + \frac{M_1(\omega)}{\omega}\}^{-1} \tag{4.3}$$

where M_1 and M_2 are the real and imaginary parts of the function M.

We have evaluated the effective mass and scattering time for (001) Si inversion layers taking into consideration the thickness effect following Stern and Howard,[8] and thus improving the previous theories. The two potentials in Eq.(4.2) are determined by the layer configuration. Figure 1 represents our theoretical curve for the effective mass.

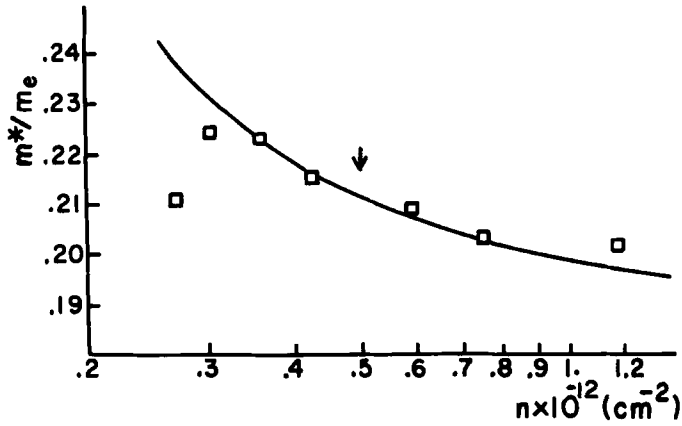

FIGURE 1 Effective mass of the electrons in Si (001) inversion layers as a function of electron density. The data represent Sample 3 of Wagner et al[9] for a magnetic field 25.4 cm^{-1}. Solid curve: Our theory. The arrow indicates a small discontinuity in slope.

The data represent Sample 3 of Wagner et al for magnetic field 25.4 cm^{-1}. For the evaluation, the impurity concentration $n_i = 1.4 \times 10^{11}$ cm^{-2} has been used. The agreement with the data is very good.

Figure 2 illustrates the scattering time. Its density dependence is opposite to that for the effective mass. As the electron density decreases, the electron-electron interaction becomes more effective, causing larger effective mass. On the other hand, the impurity scattering becomes more effective in the opposite direction. Hence, these dependences represent important many body effects. We have also succeeded in explaining the field dependencies of the effective mass and relaxation time of the 2D electrons.

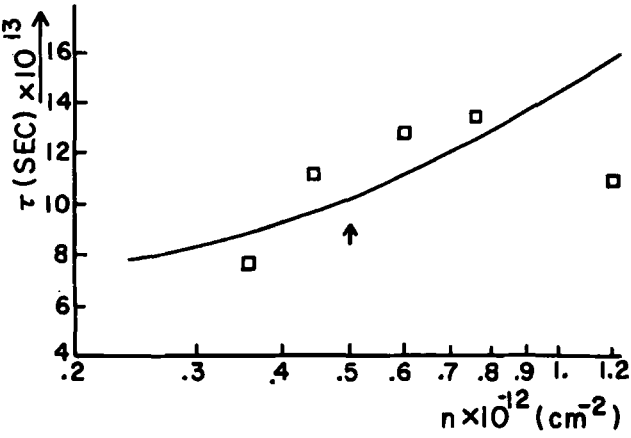

FIGURE 2 Scattering time of the electrons in Si (001) inversion layers. The data are due to Wagner et al. Solid curve: our theory.

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REFERENCES

1. Electrons and Phonons in Layered Crystal Structures (Reidel, 1979). ed. by T.J. Wieting and M. Schluter.
2. Electronic Properties of Two-Dimensional Systems (North Holland, 1980). ed. by S. Kawaji.
3. F.F. Fang and P.J. Stiles, Phys. Rev. 174, 6 (1968).
4. A. Isihara and T. Toyoda, Z.f. Phys. B 23, 389 (1976). A.J. Heiger in Chem. and Phys. of One-Dimensional Metals, ed. by H.J. Keller (Plenum Pub. 1977) p. 87. See also A.M. Afanasev and Y. Kagon, JETP 16, 1030 (1963).
5. A. Isihara, Statistical Physics (Academic Press, 1971) p. 424.
6. A. Isihara and D. Kojima, Phys. Rev. B 19, 846 (1979).
7. W. Götze and P. Wolfle, Phys. Rev. B 6, 1226 (1972), C.J. Ting, S.C. Ying and J.J. Quinn, Phys. Rev. 16, 5394 (1974). A.K. Ganguly and C.S. Ting, Phys. Rev. B 16, 3541 (1977).
8. F. Stern and W.E. Howard, Phys. Rev. 163, 816 (1967).
9. R.J. Wagner, T.A. Kennedy, and B.D. McCombe, Phys. Rev. B 22, 945 (1980).